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# Marital fertility patterns and nonmarital birth ratios: an integrated approach

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## Abstract

In a 1974 paper, Coale and Trussell described an empirical relationship between the age-specific fertility rate, the marital fertility rate, and the proportion of women with first marriages. However, their key assumption was no nonmarital fertility. This obscures the relationship between nonmarital fertility and overall fertility that distinguishes many modern Western societies from those of East Asia. Here, their equation is extended to incorporate nonmarital fertility and dual equations are derived relating age-specific fertility, marital or nonmarital fertility, proportion of women with first marriages, and the proportion of births within or outside of marriage. These equations are validated with multi-year data from countries in Europe, the USA (both African-Americans and White Americans) and Japan. They also help to illustrate the dilemma facing modern societies: between a relatively high marriage age, low nonmarital birth ratios, and high fertility, they can only accommodate two in combination.

**Keywords:** General fertility, Marital fertility, Nonmarital fertility, Marriage, Nonmarital birth ratios, Demographic transition

## Introduction

Since its origins as a science, demographers have long recognized that fertility occupies a key, if not most prominent place, in the metrics of population analysis. The various meanings and measurements of fertility across age, cohort, or marital status are well established. While the predictive patterns of fertility, especially over the long term, are still difficult (Caldwell and Schindlmayr 2003; Schneider and Gemmill 2016), historical and current data provide a wealth of information and directional indications.

At its core, fertility is based on population births; however, fertility is also intimately tied with the social structure of the population, especially units of family formation. While during certain periods of history, for example, the Baby Boom period (Van de Kaa 1987), legal marriage was the overwhelmingly dominant arrangement for having and raising children in the developed world, this has not always been the case. In more recent decades, it has become increasingly less the case in most of the countries of North America, Europe, Oceania, and increasingly Latin America.

Older models such as Coale and Trussell (1974) explained age-specific fertility solely in terms of marital fertility, but this relationship no longer holds well in multiple countries. In addition, there has been much debate over what drives the proportion of all births that are delivered outside of marriage.

This paper investigates how the factors of age-specific fertility (births per 1000 women in 5-year age groups from ages 15–19 to 40–44), marriage (measured as the proportion of women with first marriages, not accounting for divorce, in 5-year age groups from ages 15–19 to 40–44), and marital fertility (births per 1000 married women in 5-year age groups from ages 15–19 to 40–44) combine to explain the marital birth ratio (percent of all births made by women within legal marriage). While it does not explicitly claim to be able to forecast or elucidate all the various economic, cultural, or social factors that lead to more or less births within marriage as a percent of the total, it allows one to derive this information given common demographic parameters. In addition, one can use these frequently forecasted parameters to derive the expected future proportion of births within and without marriage. In the first section, the history of fertility and family formation, especially in light of the second demographic transition, is described. Second, a brief history of the mathematical relationships between fertility variables is given followed by a derivation that incorporates the marital birth ratio into a coherent framework. Finally, this relationship will be validated for a variety of countries—the USA, Japan, Switzerland, and many other European countries—throughout multiple decades of the twentieth and twenty-first century using empirical data.

### **Background and history of demographic studies of fertility and the family**

Throughout the history of demography, questions of fertility have been central and the potential impacts of relative fertility, by age, nation, ethnicity, or marital status, have been heavily researched topics. Discussions by Malthus and Verhulst (1838) on population growth from simple mathematical assumptions of exponential and logistic growth grew into more detailed research on general and total fertility rates, parity, and the new options of fertility control introduced by contraception and abortion. The first demographic transition that began in Europe with falls in first mortality and later fertility emphasized the need for demographic knowledge to understand changing populations.

Family demography, while always present during the history of demography, was developed and quantified relatively late compared to fertility analyses with quantification and modeling receiving focus during the mid-twentieth century. The family unit is typically identified as a household whose members are all related in a specific way through marriage, blood, or adoption. A brief but informative history is given by Willekens (2010). Family demography has always intersected with fertility measurements since family size distributions are largely dictated by age structure, fertility, and parity distributions in the population. However, during the second demographic transition, the diversity of family styles has been increasing (Kiernan, 2001 and Kiernan 2004) leading to a much more nuanced model of family demography where only one (or none) of the biological parents is part of the household unit. This is contrary to most earlier perspectives.

For much of the mid-twentieth century, the focus on the family often assumed that the succession of life stages predominantly dictated marriage before childbearing and the rate of the former could dictate the rate of the latter. Early work on the relationship

between age at first marriage, marital fertility, and completed family size was investigated by works such as Coale and Trussell (1974), Bumpass and Mburugu (1977), and Bumpass et al. (1978). The development of the theory of the second demographic transition (Lesthaeghe and Van de Kaa 1986; Van de Kaa 1987) characterized by a decline in both fertility and traditional marriage forced the analysis of the family and fertility to change focus to declining family sizes, declining marital fertility, and even the decline in the popularity of marriage (Kiernan 2001 and Kiernan 2004).

The decline of marriage during the second demographic transition also related to earlier research on the rise in the nonmarital birth ratio which had been previously universally low across the developed world. The ratio began to rise, particularly accelerating beginning in the 1960s (Wu 2008). While the focus of the debate over the rise in proportion of nonmarital births initially focused on the USA as well as northern Europe such as Sweden, the trend eventually encompassed much of the West. Of many surprising findings, one was that in reality, the cause of this rise was not just an increase in nonmarital fertility, which is defined as the age-specific fertility among women not within legal marriage encompassing both premarital and post-marital fertility. For example, nonmarital fertility among African-American women had peaked by the 1960s and then began to decline and stabilize though there was a moderate increase in the late 1980s and early 1990s (Teele et al. 1970; Smith et al. 1996). Despite this, the increase in the nonmarital birth ratio continued uninterrupted due to a rapid decline in African-American marital fertility and marriage rates. In contrast, the later rise in the White American nonmarital birth ratio began largely due to a steady increase in nonmarital fertility. Over the next 20 years, a rising proportion of nonmarital births spread to much of Europe as well as the other countries in North America and Oceania (Cutright and Smith 1986).

The relative strength of the causes driving the rise in the proportion of nonmarital births to the total has been a key point of debate. Originally, nonmarital fertility was often incorrectly suspected as the sole cause and target of public policy. Many such models searching for the causes of nonmarital fertility were reviewed in Freshnock and Cutright (1979). Later, a more nuanced analysis began to understand that falls in marital fertility and the decline of marriage also played prominent roles such as in Smith and Cutright (1988) and Bumpass and McLanahan (1989). Smith et al. (1996) used Das Gupta's rate decomposition method (Das Gupta, 1978) to show that African-Americans and Whites in the USA had differing causes for the increases in their nonmarital birth ratios, especially from the 1980s onwards. The increase in African-Americans was driven by the decline in the percentage of women married while the increase in Whites was driven by increases in nonmarital fertility.

In line with these discoveries, the relationship between the marriage rate in a population and nonmarital fertility became an issue of focus. The postulate of a direct relationship between the nonmarital fertility rate and the proportion of unmarried women was introduced by Gray et al. (2006). They took analyses that pointed to the importance of the proportion of married women and presented a hypothetical relationship where the change in nonmarital fertility ratio under an assumption of constant general fertility rate was directly proportional to the proportion of unmarried women in the population. This was met by some debate and response in Ermisch (2009), Martin (2009), and Wu (2009). In addition, while the assumption of constant general fertility, live births per 1000 women of reproductive age

per year, is reasonable in a short run model, it does not address the relationship between general fertility and nonmarital fertility which can be an issue given the volatility of the former over longer timescales of decades.

Concurrently, research into comparisons between the lowest low fertility in Asia, as well as a handful of similarly low fertility European countries such as Greece and Cyprus relative to most other developed Western countries, led to a surprising discovery. Notably, countries in Asia and some Eastern Mediterranean countries undergoing the general features of a second demographic transition have had much sharper drops in fertility due to their low proportions of nonmarital births (Lesthaeghe 2010). This was directly relevant to the differences in lowest low fertility in East Asia (Raymo et al. 2015) and countries in Western Europe (Kohler et al. 2002). While the second demographic transition qualitatively explains the relationships between the societal acceptance of nonmarital births and relative decline of a country's total fertility, the overall relationship has been rarely quantified in a consistent manner.

### Equations of marital and nonmarital fertility

The basic equation relating age-specific nonmarital fertility, age-specific marital fertility, and age-specific fertility is Eq. 1

$$\text{ASFR} = W(a)r_w(a) + (1-W(a))r_u(a) \quad (1)$$

where ASFR is the age-specific fertility rate,  $W(a)$  is the proportion of all women with first marriages in that age range,  $r_w(a)$  is the marital fertility rate (births to married women per 1000 married women in a given age range), and  $r_u(a)$  is the nonmarital fertility rate (births to unmarried women per 1000 unmarried women in a given age range). The marital fertility rate,  $r_w$ , was further investigated by Coale and Trussell (1974). They investigated the marital fertility rate under the context of controlled fertility by age and the relationship between marital fertility and natural fertility which is defined as the absence of purposeful fertility control, contraception, or abortion. This relationship is described in Eq. 2.

$$r_w(a) = Mn(a)e^{m\nu(a)} \quad (2)$$

Here,  $a$  is a single year age;  $n(a)$  is the natural fertility schedule;  $M$  is a normalization constant; the product  $Mn(a)$  is the natural fertility;  $\nu(a)$ , always a negative value, is the function describing the control of marital fertility by age; and  $m$  is a variable that expresses how strongly a given population applies the control of fertility in marriage. The variable  $m$  ranges from zero (no control) to higher values indicating more controlled fertility in marriage. In general, countries with a lower total fertility have higher values of  $m$ . Based on calculations from data used in this paper, almost all nations in Europe, the USA, and Japan have  $m$  values above 2.0 in the current period versus values that neared 1.0 in the immediate post-WWII years.

Coale and Trussell also described the age-specific fertility rate in terms of the proportion of women of that age with first marriages and the marital fertility. This was exactly Eq. 1 with the assumption of  $r_u(a) = 0$ .

$$\text{ASFR} = W(a)r_w(a) \quad (3)$$

Given the time period of the paper, Coale and Trussell felt confident that all fertility in a population, including nonmarital fertility, could be modeled by this equation by adjusting the marital fertility through  $m$ . Even in the face of a higher share of births out of wedlock, numerical calculations for the variables in Eq. 2 can be done understanding that their values would be blended values for the population incorporating non-negligible nonmarital fertility instead of just marital fertility.

However, neither Eqs. 1 nor 3 addresses one of the most useful demographic variables when measuring nonmarital fertility, the nonmarital birth ratio which measures the proportion of births over a period of time to mothers outside of formal marriage. Here, we will use the previous equations to derive a new relation that incorporates the nonmarital birth ratio with the other fertility variables.

The ratio of the crude birth rate of nonmarital births and marital births can be expressed using the terms on the right of Eq. 1:

$$\frac{B_u(a)}{B_w(a)} = \frac{1-W(a)}{W(a)} \frac{r_u}{r_w} \quad (4)$$

Equation 4 demonstrates that the relative values of unmarried and married births are due to the product of the inverse of the odds,  $W/1 - W$ , a woman in a certain age group has had a first marriage and the ratio of the nonmarital and marital fertility rates. The marital birth ratio,  $b_w$ , or the proportion of all births in wedlock can be derived as:

$$b_w(a) = \frac{1}{1 + \frac{B_u(a)}{B_w(a)}} \quad (5)$$

The next step is combining Eqs. 1 and 4 by taking their product as shown in Eq. 6:

$$\text{ASFR} \times \frac{B_u(a)}{B_w(a)} = (1-W(a))r_u \left(1 + \frac{1-W(a)}{W(a)} \frac{r_u}{r_w}\right) = (1-W(a))r_u \frac{1}{b_w(a)} \quad (6)$$

which by substituting Eq. 4 for  $B_u(a)/B_w(a)$  can be more concisely expressed as:

$$\text{ASFR} \times b_w(a) = W(a)r_w(a) \quad (7)$$

Given the  $a$  variable in parentheses is to remind that given these are age-specific variables, all variables in the equation must be valid within the same age range. Equation 7 and subsequent derivations are also applicable to cohort data with the requirement that  $a$  be the single year age of the women in the cohort and that all measures of fertility and marriage are restricted only to women of that cohort at a single point in time.

Though it is focused primarily on limited age ranges, for theoretical analysis, we can extend Eq. 7 to the entire population of women aged 15–44 without too much of a loss of generality to solve for the general fertility rate in terms of the same variables. This does not hold as well with empirical data due to differences of fertility control across age groups, but is still relatively accurate.

$$\text{GFR} \times b_w = W r_w \quad (8)$$

These two equations demonstrate that the age-specific fertility calculations can directly incorporate marital birth ratios with the other common variables of marital and age-specific fertility and the prevalence of marriage. Likewise, we can rearrange Eq. 7 to focus on the nonmarital fertility and the proportion of births outside of wedlock.

$$\text{ASFR} \times b_u(a) = (1 - W(a)) r_u(a) \quad (9)$$

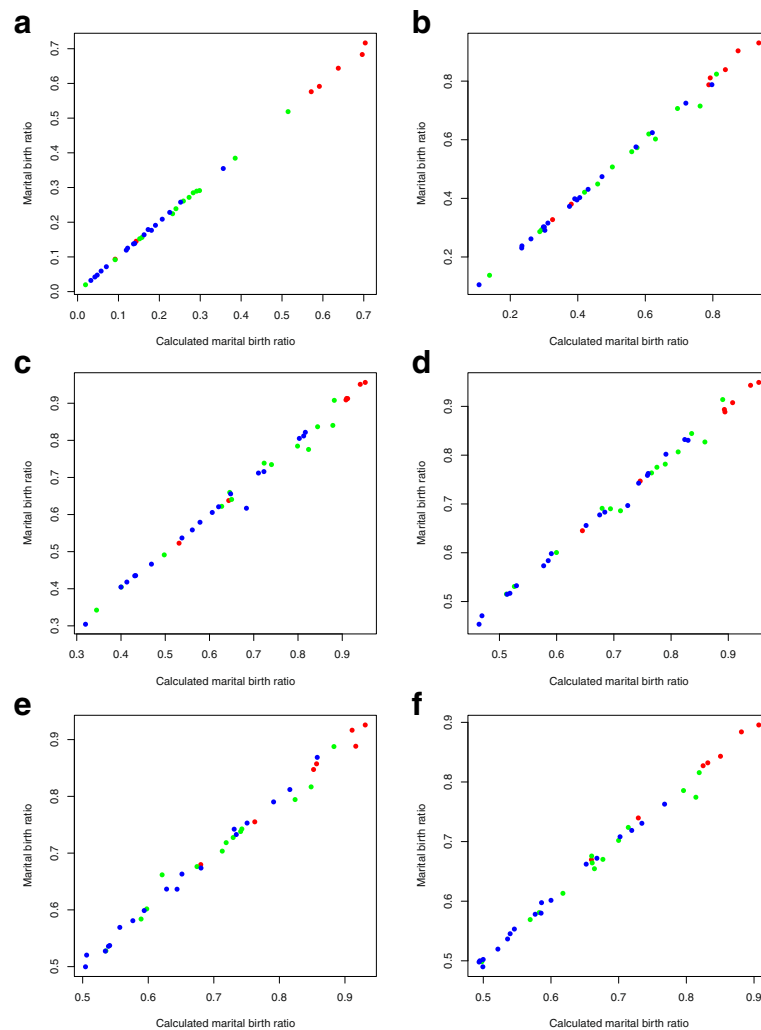
The empirical data fits well against Eqs. 7 and 9 in a variety of populations and settings. It also helps clarify some of the key questions regarding the relationships between fertility, marriage, and marital birth ratios. First, it demonstrates the inter-relationship and dual importance of both marital/nonmarital fertility and the proportion of women married. Either of these variables can be held constant, yet the other varied to produce an effect on the proportions of births in wedlock. Dropping either produces a decrease in the proportion of births in wedlock unless age-specific fertility decreases. However, Eqs. 7, 8, and 9 also allow us to analyze the effect of the changes in values of marital fertility and proportions married across different values of age-specific fertility.

Second, Eq. 9 addresses the phenomenon about the relative differences between fertility rates and nonmarital birth ratios across different contexts. In one case, the differences of nonmarital birth ratios in low fertility countries in parts of Europe and those in East Asia become clear. In Eq. 8, given marital fertility on the right-hand side of the equation, the value on the left-hand side can be any combination of values that trades off general fertility and the proportion of births in wedlock. Some countries such as Japan, Korea, and Taiwan have kept their proportion of births in wedlock very high—to the detriment of fertility—while others such as in Scandinavia and in the USA have maintained relatively higher levels of fertility while dealing with a large percentage or even a majority of births out of wedlock though this often still accommodates family formation through cohabitation (Kiernan, 2001 and Kiernan 2004).

### Validation with EU, US, Japanese, and Swiss historical data

To verify Eq. 7, historical data from several regions was obtained in order to compare the calculated value of  $b_w$ , the proportion of births within wedlock, based on the given values for ASFR, proportion of women married, and the marital fertility rate. Figure 1 shows six charts representing the actual measured and expected marital birth ratio among women in six age groups ((a) 15–19 years, (b) 20–24 years, (c) 25–29 years, (d) 30–34 years, (e) 35–39 years, and (f) 40–44 years) in multiple European countries from 1991 to 2011. As the charts show, the fit between the predicted and actual value follows a tight linear fit indicating that the calculated and actual values are nearly identical and consistent across countries and decades.

In Fig. 2, similar data, with all of the same age groups and years consolidated on a single graph, is shown for two USA populations, (a) African-American women and (b)

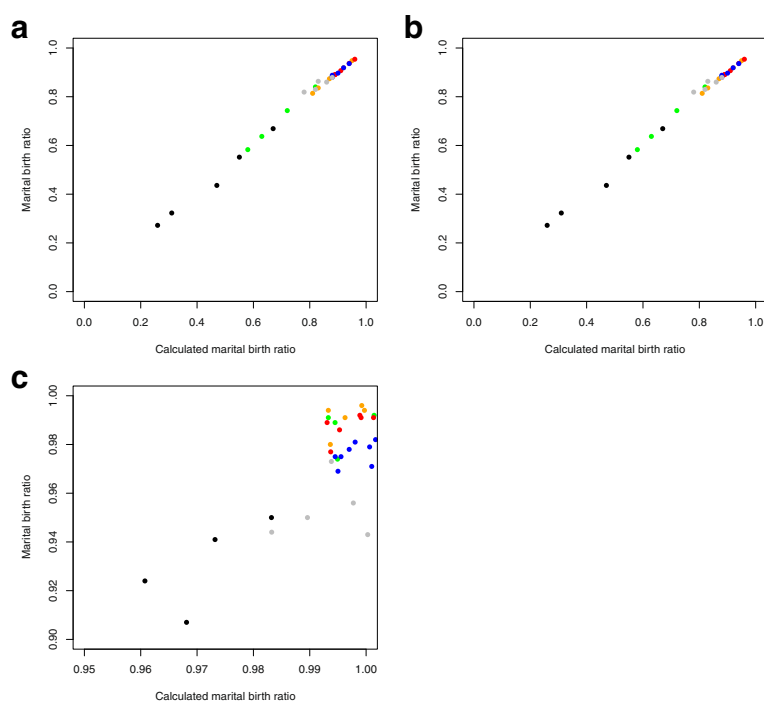


**Fig. 1** The marital birth ratio,  $b_w$ , from actual Eurostat data vs. the calculated marital birth ratio based off marriage and fertility data from Eurostat for the age groups of **a** 15–19, **b** 20–24, **c** 25–29, **d** 30–34, **e** 35–39, and **f** 40–44. Colors indicate data from the years of 1991 (red), 2001 (green), and 2011 (blue). All comparisons have  $R^2 > 0.99$ . Data from Eurostat (2018a, b, c, d, e, f), see details in [Appendix](#)

White American women, as well as (c) Japanese women. The first two plots are African-American women and White American women respectively from 1980 to 2000 by 5-year increments. The third is Japanese women from 1950 to 2010. US African-American and White data fit relatively well similar to European data. The main outlier seems to be the 40–44 age group of African-American women; however, it seems this is due to the quality of the data rather than model issues.

In the 1980s and even later, there is a widely known sampling error and undercount bias that has been recognized by the Census of the African-American population Robinson et al. (1993). This is especially pronounced at the younger ages though it affects the earlier ages as well. Robinson et al. (1993) indicate that for the 1990 Census, Black women in the 40–44 age group are least undercounted percentage-wise (1.5%), whereas Black women 25–29 have the highest undercount (4.9%). However, the relatively small





**Fig. 2** The marital birth ratio,  $b_w$ , vs. the calculated marital birth ratio based off marriage and fertility data for all 5-year age groups and years of 1980, 1985, 1990, 1995, and 2000 for two US populations, **a** African-American women and **b** White American women. Colors of points indicate the age groups of 15–19 (black), 20–24 (green), 25–29 (orange), 30–34 (red), 35–39 (blue), and 40–44 (gray).  $R^2 > 0.99$  for White American women of all ages and African American women minus the 40–44 age group. Inclusion of the possibly erroneous 40–44 age group lowers  $R^2$  to 0.73 for African-American women. Note that while  $R^2$  is high for these data as well, they are not directly comparable to Fig. 1 due to aggregation of age groups in Fig. 2 (due to less data) versus a separation of age groups in Fig. 1. They demonstrate that the same relationship holds however both within and between age groups. Data from Ventura and Bachrach (2000); Martin et al. (2002); National Center for Health Statistics (NCHS) (2006); Martin et al. (2017)). In **c**, a similar analysis is done for Japanese women with data from 1950, 1960, 1970, 1980, 1990, 1995, 2000, 2005, and 2010 with the same colors representing age brackets as the US populations. Data fits an  $R^2$  of 0.36. Data from National Institute of Population and Social Security Research (NIPSSR) (2012). See details in Appendix

number of births in the 40–44 age group likely skews fertility rate calculations. For example, the marriage proportion could not be directly calculated for 1995 because the nonmarital fertility rate was identical to the ASFR at 6.0. This can only happen under the circumstance that both the marital and nonmarital fertility rates are equal, which they are not because the marital fertility is given at 6.4, or if the proportion of Black women aged 40–44 who are married is zero, which is false. The marriage ratio for 1995 was thus estimated as the midpoint of the 1990 and 2000 values.

Japanese women show the least consistent fit of data to the model, partially because its marital birth ratio is almost uniformly high above 97% across all ages over the time period measured. This relatively narrow range seems to give fluctuations from “noise” or deviations from the model fit a larger impact than the wider ranges of marital birth proportions in the USA and EU and thus a much lower  $R^2$  than the other data sets.



Finally, Fig. 3 demonstrates the previous relationship from Figs. 1 and 2 but over a longer time period than other European countries using Swiss data from the 1940s to the 1990s (Calot et al. 1998). This data is included to show that the relationship is not just a contemporary one from the last few decades of societal change but fits over all demographic situations where data can be measured. Detailed information on the data used for Figs. 1, 2 and 3 is available in the Appendix.

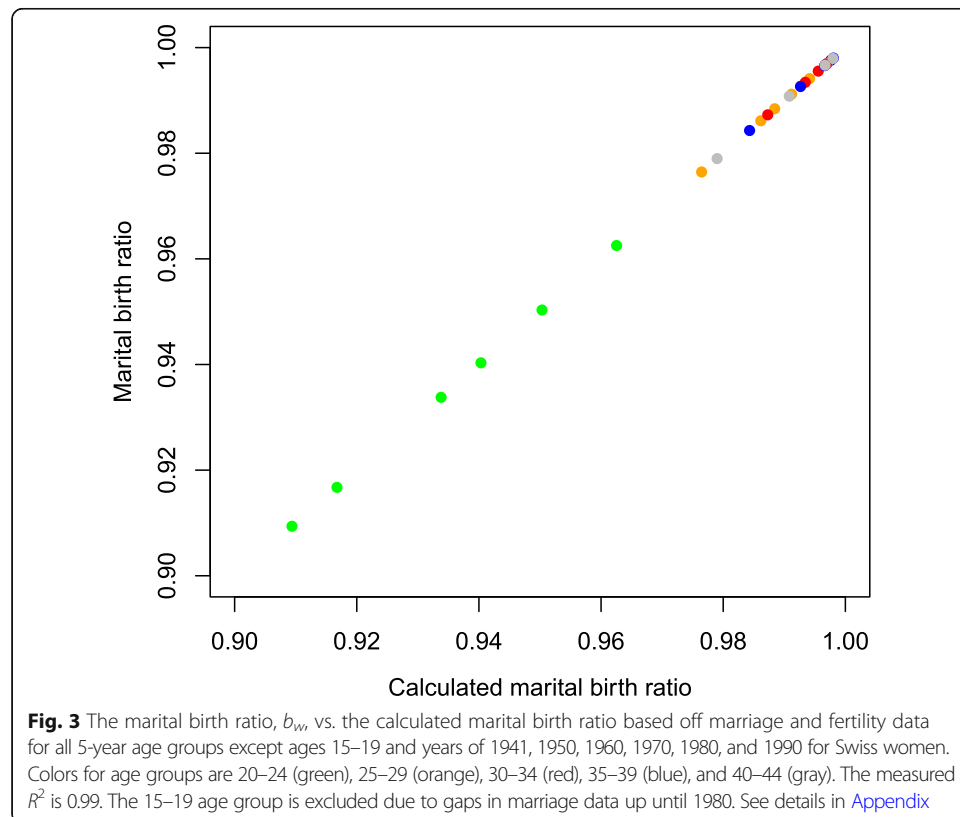
### Differential changes in marital birth ratios

To investigate the relative effects of the causes in changes of the variables on the marital birth ratios, the Eq. 10 suffices.

$$\Delta b_w(a) = \frac{1}{ASFR} \left[ r_w(a)\Delta W(a) + W(a)\Delta r_w(a) - \frac{W(a)r_w(a)}{ASFR} \Delta ASFR \right] \tag{10}$$

The final term in the brackets can be dropped if age-specific fertility is assumed to change relatively slow or not at all. What is interesting is even under level fertility, age-specific fertility still plays a role and reduces changes in the marital birth ratio depending on its value. In a society where the relative proportion of births within and without of wedlock is relatively constant, Eq. 10 can be reduced to:

$$\frac{\Delta W(a)}{W(a)} + \frac{\Delta r_w(a)}{r_w(a)} - \frac{\Delta ASFR}{ASFR} = 0 \tag{11}$$



$$\frac{\Delta \text{ASFR}}{\text{ASFR}} = \frac{\Delta W(a)}{W(a)} + \frac{\Delta r_w(a)}{r_w(a)} \quad (12)$$

Thus, the percentage change in the age-specific fertility equals the sum of the percentage change in the proportion married plus the percentage change in marital fertility. This condition has usually not held in other countries over recent years excepting a few such as Japan, South Korea, and Israel (OECD 2017) and the Republic of China (Taiwan) (Republic of China, Ministry of the Interior, 2016). Table 1 shows the data reflecting this from the 25–29 age group in Japan over the time frame 1950 to 2010 where the proportion of births outside of wedlock has hardly changed.

If the marriage rates are also relatively constant and applying the same derivation using the differential version of Eq. 9, we can conclude:

$$\frac{\Delta \text{ASFR}}{\text{ASFR}} = \frac{\Delta r_w}{r_w} = \frac{\Delta r_u}{r_u} \quad (13)$$

So under these conditions, the percentage changes in the marital and nonmarital fertility are equal to each other and the change in age-specific fertility. This has not been the case in most societies undergoing second demographic transitions, however, since the decline of marriage has been a general feature.

### Analyzing the effect of marriage under constant age-specific fertility

After discussing the relationships between age-specific fertility, marriage, and the marital birth ratio, it can be instructive to analyze how different marital behaviors in a population can influence the marital birth ratio. This is especially cogent in examining the possible futures for family structures given a desired total fertility rate (for example, total fertility of 2.1 or higher).

Given the total fertility rate is the sum of the product of the 5-year age-specific fertility rates times 5 and divided by 1000, if one assumes each age group proportionally contributes to fertility the same way over a variety of total fertility scenarios, age-specific fertility can be estimated and used to estimate family

**Table 1** Data from the 25–29 age group in Japan over the time frame 1950 to 2010

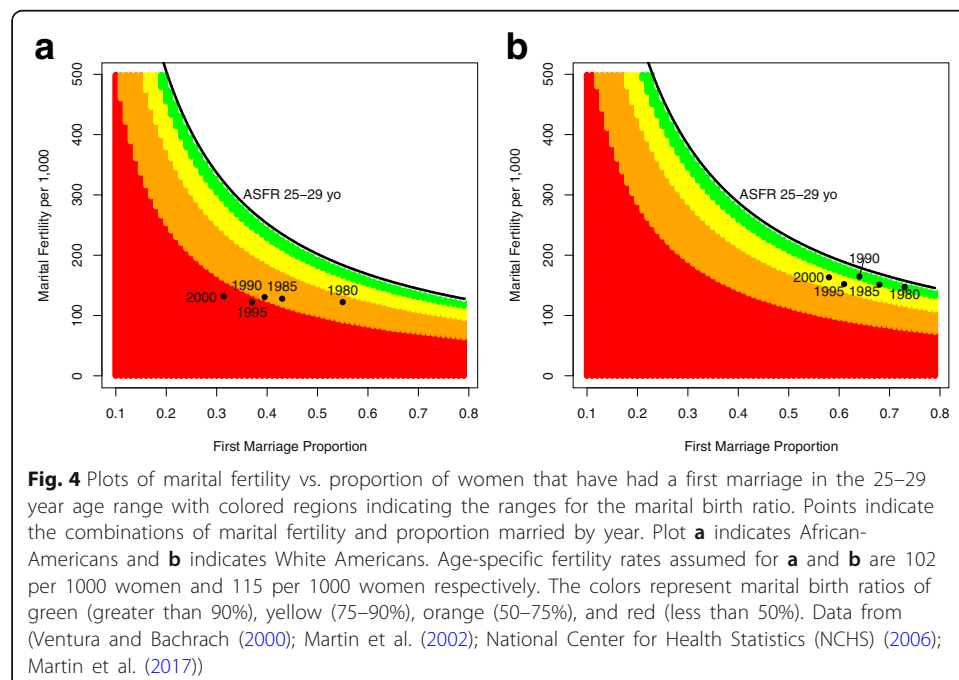
Year	$\Delta \text{ASFR}/\text{ASFR}$	$\Delta r_w/r_w$	$\Delta W/W$	$\Delta r_w/r_w + \Delta W/W$
1960	–0.23	–0.21	–0.03	–0.24
1970	0.15	0.09	0.05	0.14
1980	–0.13	–0.06	–0.07	–0.13
1990	–0.23	–0.01	–0.22	–0.23
1995	–0.17	–0.03	–0.14	–0.17
2000	–0.14	–0.02	–0.13	–0.14
2005	–0.14	–0.02	–0.12	–0.14
2010	0.02	0.06	–0.03	0.03

Legend: Eq. 12 demonstrated for Japanese women ages 25–29 from 1950 to 2010. National Institute of Population and Social Security Research (NIPSSR) (2012)

structure scenarios (at childbirth). Using Eq. 7 and assuming a constant ASFR, a graph of the age-specific marital fertility ( $r_w$ ) vs. the proportion of women with a first marriage ( $W$ ) can delineate the regions of various marital birth ratios ( $b_w$ ) in a contour plot.

In Fig. 4, two graphs demonstrate this for the (a) African-American and (b) White American populations from 1980 to 2000 where the total fertility rates (TFR) were relatively constant (TFR of 2.1 to 2.4 for African-Americans and TFR of 1.7 to 2.0 for White Americans; the peaks for both were only in the early 1990s) and along with the age-specific fertility rates for 25–29 year olds (100 to 115 per 1000 women, averaging near 102, for African-Americans and 100 to 120 per 1000 women, averaging near 115, for White Americans). The colored regions indicate different ranges for marital birth ratios, green indicating greater than 90%, 75–90% for yellow, 50–75% for orange, and less than 50%. These colored regions are determined using the calculations of Eq. 4 with the assumed fixed age-specific fertility rate.

The black border line indicates the frontier of the plot where 100% of births are within marriage and the marital fertility rate times the proportion of women having a first marriage equals the age-specific fertility rate, the exact relationship first postulated in Eq. 3 by Coale and Trussell (1974). The points indicated by years display the values of  $W$  and  $r_w$  by year and how this matches with the expected values of  $b_w$ . One can see the relative effects of the reduction in the proportion of women married and the reduction in marital fertility and how it affects the ratio of births in wedlock. Similar plots can be made for population scenarios, using forecasted or targeted fertility rates, to understand how different scenarios of family formation will affect the ratio of births within and outside of wedlock.



## Conclusion

There is a tight interlocking relationship between fertility, the prevalence of marriage, and the proportion of unwed births. However, this relationship only connects the variables and does not specify the causes nor the forces that shift demographic parameters in a population. The forces affecting the popularity of the institution of marriage as well as the determinants of married and population-wide fertility are complex and multifaceted (Bongaarts 1978). These equations do not claim to forecast or even determine what those rates can or will be in the future. In fact, at least three of the variables are free to take a variety of values, only the fourth and final one are necessarily fixed.

The force of the equations is to help quantify the effects of the second demographic transition in societies and to show how different societies facing the same forces, such as declining marriage and marital fertility, can respond differently in relation to age-specific fertility and nonmarital births. They can also be combined with techniques such as rate decomposition from Das Gupta (1978) to fully understand what drives the different demographic fertility indicators across times or societies.

While the equations themselves impose no constraints, they can demonstrate constraints placed on demographic trajectories by trends in populations. For example, for all age groups, the proportion of women married negatively correlates with mean or median age of first marriage.

The proportion of women married in an age group is directly dependent on the average and median age of first marriage. The (first) marriage frequency and the risk function of being married at a given age fit common statistical distributions as described in Coale (1971), Coale and McNeil (1972), and Kaneko (2003). All of these distributions will have cumulative distributions that shift or change shape to reflect a lower proportion of those ever married in lower age groups due to an increased mean or median marriage age. This is not even including the effects of an increasing number of men and women who never marry.

Marital fertility can also tend to decrease as average or median marriage age increases, but the causation is not as clear and there are exceptions such as increased marital fertility at the highest age groups such as 40–44 as marriage is postponed later in life. Given these dependencies, a rising marriage age typically lowers the right side of the equation and constrains the product of the terms on the left: the age-specific or general fertility rate and the proportion of births in wedlock. This puts modern societies that have undergone the second demographic transition in a bit of a quandary: between a relatively high marriage age, low nonmarital birth ratios, and high (hopefully replacement) fertility, they must pick only two.

In conclusion, while the analysis in this article cannot fully articulate the underlying drivers behind societal changes such as decline in marriage or lower total fertility, it can explain how the effects of the lower rates of marriage and higher ratios of nonmarital births integrate with overall societal fertility in order to understand the effect of forecasted changes or analyze scenarios regarding the demographic development of societies affected by the second demographic transition.

## Appendix

Data background on countries and years used in Figs. 1, 2, and 3

### Eurostat data

“All years” indicates full data was available for 1991, 2001, and 2011. Note Liechtenstein did not provide data on the 15–19 age group.

	15–19	20–24	25–29	30–34	35–39	40–44
Belgium	All years	All years	All years	All years	All years	All years
Czech Republic	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Denmark	2011	2011	2011	2011	2011	2011
Finland	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
France	2011	2011	2011	2011	2011	2011
Germany	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Hungary	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Iceland	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Italy	1991	1991	1991	1991	1991	1991
Latvia	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Liechtenstein	N/A	2011	2011	2011	2011	2011
Lithuania	2011	2011	2011	2011	2011	2011
Luxembourg	1991	1991	1991	1991	1991	1991
Netherlands	All years	All years	All years	All years	All years	All years
Norway	All years	All years	All years	All years	All years	All years
Romania	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Slovakia	2011	2011	2011	2011	2011	2011
Slovenia	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011	2001 and 2011
Sweden	All years	All years	All years	All years	All years	All years
Switzerland	All years	All years	All years	All years	All years	All years

Background on data from Eurostat (2018a, b, c, d, e, f): data for the proportion of women married was calculated using the married women population by age data from the table [demo\_pjanmarsta] and the total population in women by age from table [cens\_hnmga]. Marital fertility was calculated using the live births to married women in table [demo\_fagec] combined with the population in married women from [demo\_pjanmarsta]. Age-specific fertility rates and the marital birth proportion were obtained from tables [demo\_frate] and [demo\_fagec] respectively.

### US data

For both African-American women and White American women, the years 1980, 1985, 1990, 1995, and 2000 were used. All data was obtained from Ventura and Bachrach (2000), Martin et al. (2002); National Center for Health Statistics (NCHS) (2006), Martin et al. (2017) with the exception of proportions of women married. This data was not consistently available from government statistics by age group so was calculated using Eq. 1 and data on marital and nonmarital fertility rates along with age-specific fertility rates. The marriage proportion for African-American women 40–44 in 1995 was estimated as the midpoint of the 1990 and 2000 values as described in the text.

### Japan data

All data for Japanese women was obtained from National Institute of Population and Social Security Research (NIPSSR) (2012) for the years 1950, 1960, 1970, 1980, 1990, 1995, 2000, 2005, and 2010.

### Swiss data

Data for Swiss women was available for the years 1941, 1950, 1960, 1970, 1980, and 1990 in Calot et al. (1998). Their data set includes the proportion of women with first marriages by single age years, the age-specific fertility rate for single ages and 5-year age groups, the age-specific nonmarital fertility for single ages, and 5-year age groups and the female population by age. These variables were used to derive the nonmarital birth ratios and marital fertility for calculations.

### Abbreviations

ASFR: Age-specific fertility rate (5 years; per 1000 women);  $B_u(a)$ : Crude nonmarital birth rate (5 years);  $b_u(a)$ : Proportion of all births to unmarried women (5 year);  $B_w(a)$ : Crude marital birth rate (5 years);  $b_w(a)$ : Proportion of all births to married women (5 years); GFR: General fertility rate;  $r_u(a)$ : Nonmarital fertility rate (per 1000 unmarried women) in 5 year age range  $a$ ;  $r_w(a)$ : Marital fertility rate (per 1000 married women) in 5 year age range  $a$ ; TFR: Total fertility rate;  $W(a)$ : Proportion of women having had a first marriage in 5 year age range  $a$

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### Availability of data and materials

Smith, Reginald. (2018). Marriage Proportion, Age Specific Fertility, Births within Marriage ratios for US, Japan, and selected European countries (Version 1.0) [Data set]. Zenodo. <https://doi.org/10.5281/zenodo.1219745>

### Author's contributions

The author read and approved the final manuscript.

### Ethics approval and consent to participate

Not applicable.

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