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Why life expectancy over-predicts crude death rate

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Abstract

Life expectancy is widely used and reported; for example, the UN Human Development Index uses life expectancy as a key component. But many users of period life expectancy do not understand and interpret life expectancy as demographers do. In particular, period life expectancy almost always over-predicts the crude death rate. Indeed, in most observed populations, the annual deaths recorded are always less than one may expect in the corresponding stationary population. To explain this over-prediction, in this paper we analyze how deviations from stationarity affect the crude death rate. We use theory to show that small deviations from the stationary age structure (as high as 20% at each age) always lead to over-prediction. Then we examine global data to show that over-prediction is widespread, occurring even in populations where the deviation from stationarity is large. Finally, we show that populations around the world and over many decades have age structures that are almost always far from stationary (or indeed, stability). But we also show that the deviation is often due to the demographic transition, with population bulges at the middle ages where mortality is low.

Keywords: Sensitivity, Crude death rate, Life expectancy

Introduction

Life expectancy is a widely reported statistic, for example by official governmental agencies every year, or various United Nations agencies such as the Human Development Program (United Nations Development Programme, 2022). However, a common (but incorrect) interpretation of period life expectancy e_0 is that it measures actual lifespan in that period's population. In other words, the number of deaths in an observed population should be (total population)$/e_0$. But in several countries, the actual number of deaths is much smaller than the ratio would predict. For example, in the United States with a population of 337 million and a life expectancy at birth of 77 years, we should expect an annual number of deaths of $(337/77) = 4.38$ million. But the actual number is about 3.28 million (Department of Economic United Nations & Population Division Social Affairs, 2022). Such a large over-prediction is surprising (at least to us).

Mere inequality is of course no surprise to a demographer—as the latter would say, only in a "stationary" population is the average age at death given by e_0, and only in a stationary population do deaths equal (total population)$/e_0$. That corresponding stationary population is of course defined by having zero growth and the mortality rates of an observed population. In an observed population, the number of deaths depends on the actual age structure, not on a hypothetical "stationary" age structure. One can readily imagine population structures where the number of deaths is much higher (e.g., a population with everyone over 70) or much lower (e.g., a population where everyone is between 15 and 25) than in a stationary population. But such examples are unrealistic and they also cut both ways, in the sense of implying either higher or lower actual deaths. Here we answer the following questions that grow out of the above observations. Why do we so easily find cases where the ratio (total population)$/e_0$ is greater than the number of deaths? Is the over-prediction typical, and if so, why? What do we find in a global sample, using the United Nations World Population Prospects (hereafter, WPP) (Department of Economic United Nations & Population Division Social Affairs, 2022), where data cover a wide range but data quality is uneven? What do we find in high-quality data, as in the Human Mortality Database (hereafter HMD) (Berkeley USA University of California & Max Planck Institute For Germany, 2022)? Does the pattern of age-structures change in a common way across countries, and if so why?

It is obvious that crude death rate depends on the age-patterns of both mortality and population, but how? We derive an equation that reveals how these two factors determine the difference between the actual crude death rate and the stationary crude death rate. For small perturbations, we obtain a sensitivity for the crude death rate. Our sensitivity has a pleasing mathematical similarity to the famous Keyfitz (1977) and Demetrius (1978) "entropy"-like sensitivity of life expectancy. As in the latter case (Vaupel, 1986), we show that our result reveals the age-specific sensitivity of crude death rate to change in the population age structure, and that the sensitivity differs in high and low mortality contexts. Then we use our analysis to show that small deviations in age-structure from stationarity always lead to over-prediction. Our result suggests that the extent of over-prediction depends on the size of the deviation from stationarity.

Next we explore crude death rates across many countries and time periods, using data from both HMD and WPP to compute the difference between crude death rate and $(1/e_0)$. We find a striking global over-prediction of the crude death rate. Then we use long-term Swedish data to provide a useful perspective on the crude death rate over a long period with distinct demographic regimes. Of course, over-prediction does not always occur, and we show when under-prediction occurs and discuss reasons why.

Finally, we explore the deviation from stationarity in populations across many countries and periods. We use data from the WPP and the HMD to show that the age-structural deviation from stationarity is usually small enough that over-prediction will almost always be found. Then we examine more closely how the deviation from stationarity changes over time. Using examples, we argue that the demographic transition produced deviations in the form of population bulges that have always led to what we call over-prediction. We also provide a longer-term perspective by examining age-structural change in Sweden from 1751 to 2021. We conclude with a brief discussion.

Crude death rate and age structure

Consider a population (in a specified year) in which individuals at age x have a death rate $\mu(x)$ and a probability $l(x)$ of surviving to at least age x. We consider only females here; two sexes can be included with more algebra. Say that the population's age-structure is described by a fraction $u(x)$ of individuals at age x. The per-capita death rate is called (by demographers) the crude death rate, and is

$$C_D = \text{deaths per capita} = \int dx \mu(x) u(x). \tag{1}$$

We use a per-capita rate so the results apply for any total population.

In a stationary population with the same age-specific mortality rates, the fraction $u_s(x)$ of individuals at age x is $(l(x)/e_0)$, and the crude death rate is

$$C_{Ds} = \left(\frac{1}{e_0}\right). \tag{2}$$

(The above follows from (1), using $l(0) = 1$). How do these two crude rates compare?

Sensitivity of crude death rate

Say that an observed population has a non-stationary age structure:

$$u(x) = [u_s(x)]^{(1+g(x))}. \tag{3}$$

Then Eq. (1) implies that the crude death rate is

$$C_D = \int dx\, \mu(x)\, [u_s(x)]^{(1+g(x))}. \tag{4}$$

Here and later the lower limit on the integral is always zero; the upper limit may be taken as infinity, or some finite number that exceeds the largest age at death. The difference between the crude death rates in an actual and a corresponding stationary population is

$$\delta C_D = \left[C_D - \left(\frac{1}{e_0}\right)\right]. \tag{5}$$

This difference is a *sensitivity* when $g(x)$ is small.

Take $g(x)$ to be small enough so that we can take logarithms in (3) and expand to get:

$$[u_s(x)]^{(1+g(x))} \simeq u_s(x) + g(x) u_s(x) log u_s(x). \tag{6}$$

This is of course only a linear expansion, but later numerical work shows that the approximation is reasonable for $g(x)$ as large as say 0.2, which corresponds to a 20% deviation from stationary age structure. Using this expansion in (5), the sensitivity of the crude death rate is thus

$$\delta C_D = C_D - \left(\frac{1}{e_0}\right) \simeq \int dx g(x)\, \mu(x)\, u_s(x) \log u_s(x) \tag{7}$$

$$= \int dx\, g(x)\, \mu(x) \left(\frac{l(x)}{e_0}\right) \log\left(\frac{l(x)}{e_0}\right). \tag{8}$$

As we next show, the above expression quantifies the intuition that the actual value of sensitivity depends on the age pattern of the deviations $g(x)$.

Sensitivity and entropy-like measures

The sensitivity (8) shows that the contribution of a deviation in the age-structure at age x is proportional to the ratio:

$$-\mu(x)\left(\frac{l(x)}{e_0}\right) log\left(\frac{l(x)}{e_0}\right). \tag{9}$$

We use the minus sign because the logarithm is always negative. Thus a positive deviation at any age, $g(x) > 0$, will decrease δC_D, and the converse.

There is a striking similarity between the sensitivity in (8) and the sensitivity of life expectancy to mortality change (Demetrius, 1978; Goldman & Lord, 1986; Keyfitz, 1977; Vaupel, 1986). In the latter, $g(x)$ is a small fractional change in mortality rate at age x, and the resulting change in life expectancy is

$$\delta e_0 = \int dx\, g(x)\, l(x)\, \log l(x)\, . \tag{10}$$

Thus the contribution of a change at age x to δe_0 is proportional to the ratio

$$-l(x)\, log l(x), \tag{11}$$

as discussed by Vaupel (1986). If we ignore the mortality factor in (9), the age-pattern of the remaining contribution to δC_D is similar to that of δe_0. As we now show, the mortality factor in (9) does not alter the essential pattern.

Figure 1 shows the age-specific sensitivity weights from Eq. (9) for Sweden in four different years (data from the HMD). In high-mortality regimes (the earliest year, 1850) with high infant mortality and old-age mortality, the maximum value of sensitivity weight (0.013, 12.6% of the overall weight) occurs at age zero. With the decline in infant mortality (1950), the sensitivity weight at age zero drops dramatically and the maximum value shifts to age 80. As the mortality rate decreases further, the maximum value moves to age 87 and age 90 in 2000 and 2019, respectively. This figure supports demographic intuition, and shows that a decrease in the proportion of very young and/or old people will lead to a high positive δC_D. But do we often see such an age pattern, or the opposite, and why?

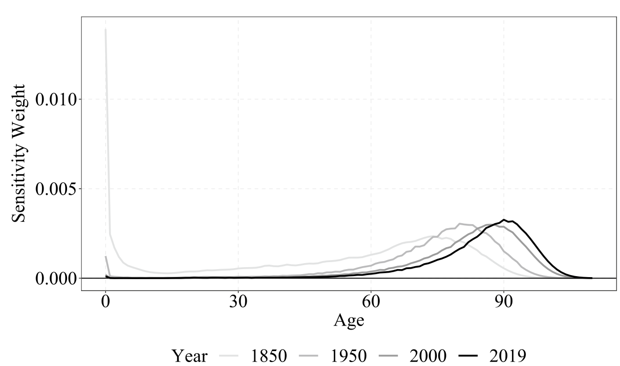

Fig. 1 Sensitivity Weight in Sweden from HMD

Sensitivity and the Kullback–Leibler distance

We now focus on the age-structure of the population. In the definition of $g(x)$ in (3), take logarithms on both sides to see that

$$g(x)\, log u_s(x) = log\left(\frac{u(x)}{u_s(x)}\right). \tag{12}$$

Now insert this in (8) to get our first expression for sensitivity:

$$\delta C_D = \int dx\, \mu(x)\, u_s(x) \log\left(\frac{u(x)}{u_s(x)}\right). \tag{13}$$

The integrand on the right measures the difference between the real age-structure $u(x)$ and the stationary structure $u_s(x)$ for a given age x. The overall departure from stationarity can be measured by the similar Kullback–Leibler distance (Kullback & Leibler, 1951):

$$K_u = -\int dx\;\, u_s(x) \log\left(\frac{u(x)}{u_s(x)}\right) \geq 0\,. \tag{14}$$

The inequality on the right above is well known.

Given that mortality rates are bounded between some $A > 0$ and some $B > 0$, the sensitivity in Eq. (13) is bounded between two negative numbers:

$$-A\, K_u > \int dx\, \mu(x)\, u_s(x) \log\left(\frac{u(x)}{u_s(x)}\right) > -B\, K_u\,. \tag{15}$$

Thus we have a key result: when a population has a structure slightly different from stationary, we always have an over-estimation:

$$\text{Crude death rate } C_D < \left(\frac{1}{e_0}\right) = C_{Ds}.$$

And from (14, 15), the over-estimation should increase with the Kullback–Leibler distance.

Observed crude death rates

We have shown mathematically that over-prediction is typical in a population with about 20% deviation from stationary. But the deviations in observed populations could be larger, or different. How typical is over-prediction in populations across time and space?

We find, as shown in Fig. 2, that most populations display the pattern we expect: negative sensitivity and over-prediction. The difference between the actual crude death rate (C_D) and ($1/e_0$) is negative in most populations and years in the HMD (94.54%) and in the WPP (95.41%).

An interesting feature of this comparison (Fig. 2) is that under-prediction (i.e., $C_D < (1/e_0)$) typically occurs when life expectancy is very high or very low relative to the average. In countries with low e_0, we likely have limited or poor data on population at old ages. In countries with high e_0, we likely have populations with long-term low fertility and thus a historically unusual age structures, dominated by an increasing percentage of older individuals. Clearly, over-prediction will not be seen in such populations.

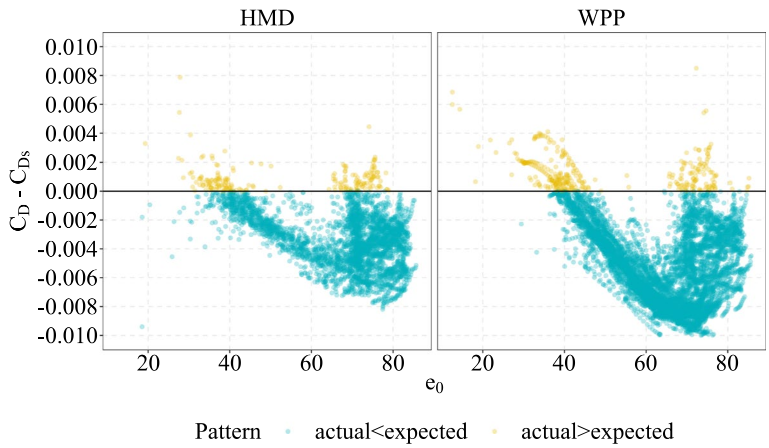

Fig. 2 The sensitivity of crude death rate (C_D—C_{Ds}) and life expectancy at birth (e_0) in countries from HMD and WPP

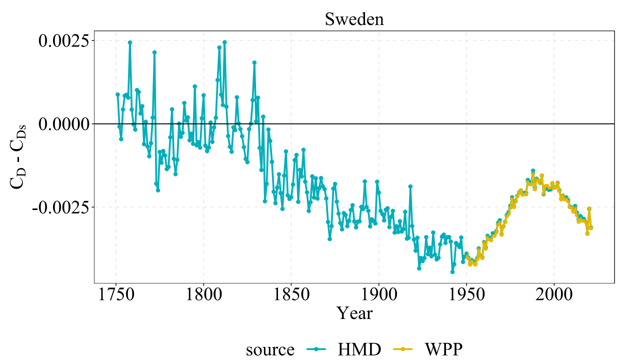

Fig. 3 The sensitivity of crude death rate (C_D—C_{Ds}) in Sweden from HMD and WPP, 1751–2021

Figure 3 reveals the long-term pattern of the sensitivity of crude death rate in Sweden. In the pre-demographic transition stage (until about 1850) we expect the population to fluctuate around stationarity, and indeed the difference between actual and stationary crude death rates fluctuates around 0. During the demographic transition (beginning in 1850 but most pronounced in the period 1900–1950) the differences become negative and decrease to a low around 1950 and then increase. Additional perspective on these changes is provided below, when we examine the changing pattern of age structure.

Changing age structures: patterns and causes

The deviation in age-structure is key to the crude death rate, but how far are populations from their corresponding stationary structures? To answer this question, we again use annual data from the HMD for 40 countries over a range of years (the longest period is 1751–2021 for Sweden). We also examined a larger set of data for the period 1950–2021 from the WPP for 126 countries and areas whose populations were over 5 million. Smaller countries were omitted as too sensitive to fluctuations.

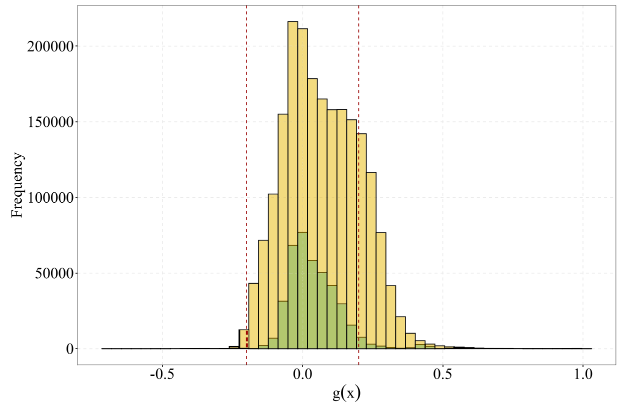

Fig. 4 Frequency of g(x) value in countries from HMD and WPP. Dashed lines are at values ± 0.2; HMD data in green and WPP data in yellow

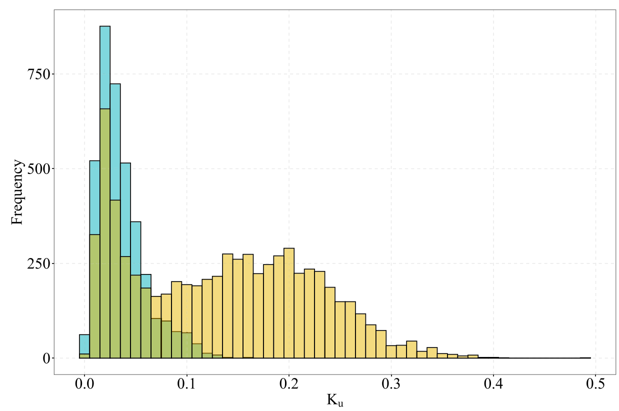

Fig. 5 Frequency of K_u value in countries from HMD and WPP. HMD data in green (or blue) and WPP data in yellow

Age-structural deviations

Recall that we compare a population's actual age structure $u(x)$, and the corresponding stationary structure $u_s(x)$. These yield the proportional deviations $g(x)$ (defined in (3)). They also yield the observed crude death rate C_D and the crude death rate C_{Ds} for the corresponding stationary population with the same death rates.

Using the data sources above, we compute and present the frequency distribution of the $g(x)$ values in Fig. 4. The peak values of $g(x)$ occur near but slightly above 0, and most of values of $g(x)$ are under 20% (about 8 in 10 for the WPP data, and about 9.5 in 10 for the HMD data). Hence age-specific deviations in observed populations from stationarity are often small enough that over-prediction will be found.

Overall deviation

We used the Kullback–Leibler distance (K_u) to measure the overall deviation in the age distribution relative to the stationary distribution. Figure 5 shows the frequency and distribution of the K_u values. All the K_u values observed here are (naturally) positive, with peaks at 0.02 and means at 0.04 (HMD) and 0.13 (WPP). Therefore, observed age

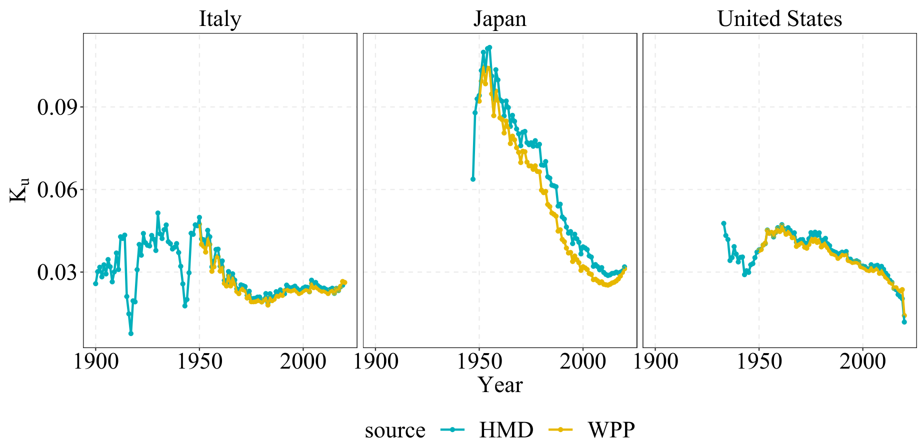

Fig. 6 K_u value in 3 countries from HMD and WPP, 1900–2020

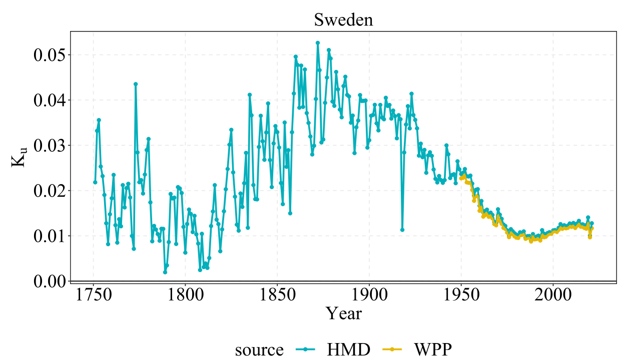

Fig. 7 K_u value in Sweden from HMD and WPP, 1751–2021

distributions are almost always non-stationary. Given that WPP data cover a wider range and are of uneven quality, we expect and find that WPP data show a wider range than HMD data, ranging from 0 to 0.5.

Note that qualitatively similar results are obtained using an alternative indicator:

$$S_u = \int dx \,(\, u(x) \; - u_s(x)\,)^2\,, \tag{16}$$

and are in the Appendix (Figures 10, 11, 12, 13).

Trends in overall deviation

To capture trends, we first present the K_u values for three countries with high-quality data in Europe, Asia, and North America from 1900 to 2020 (Fig. 6). Taking Italy (left panel) as an example, if we ignore the wide swings due to the 1918 influenza epidemic and the two World Wars (Glei et al., 2015), K_u values rose from 1900 to 1950 and steadily declined thereafter, stabilizing at levels varying between 0.02 and 0.03 after 1975. Japan and the United States follow a similar trajectory, as do many other industrialized countries.

For a long-term perspective, consider Sweden (Fig. 7) with data starting in 1751. We ignore noise in the early historical data (especially for years preceding 1870–80) caused by lower data quality (Barbieri et al., 2015). In the years before the Industrial Revolution (1750–1850), age structure fluctuated with no systematic trend, as seen in the K_u values in Sweden for that period. As industrialization took hold, and the demographic transition began, the distance K_u first rose till about 1900, and then fell steadily until leveling off at a small value after about 2000.

Bulges in age distribution

The distance K_u measures the deviation across the entire age distribution, but does not provide age-specific information (e.g., which distribution at a certain age is lower or higher). So here we compare the observed and stationary age distributions in a more detailed way by plotting their ratio.

Start with Denmark (Fig. 8, top-left panel). In the pre-demographic transition stage in which both birth and death rates are high (1850), the ratios decreased with age, falling below 1 at age 35, so the actual population was "younger" than the stationary population. In the early stage the demographic transition (1950) resulted in a bulge at ages 0–5. In later years that bulge travels to later ages, whereas there is a continuing reduction in birth rates and thus in the percentage as at young ages.

Those changing depressions and bulges are observed in many other industrialized countries, as seen in the figure. A striking common feature in those bulges is that they are concentrated in the age range 10–50, which typically is the lowest mortality range. In other words, there is a larger proportion of individuals with lower mortality in the observed population than that in the stationary population. This explains the over-prediction.

In many countries, the above temporal patterns lead to a negative correlation between deviation and sensitivity, as shown in Fig. 9. What this means is that the difference between the actual crude death rate and the stationary value becomes more negative

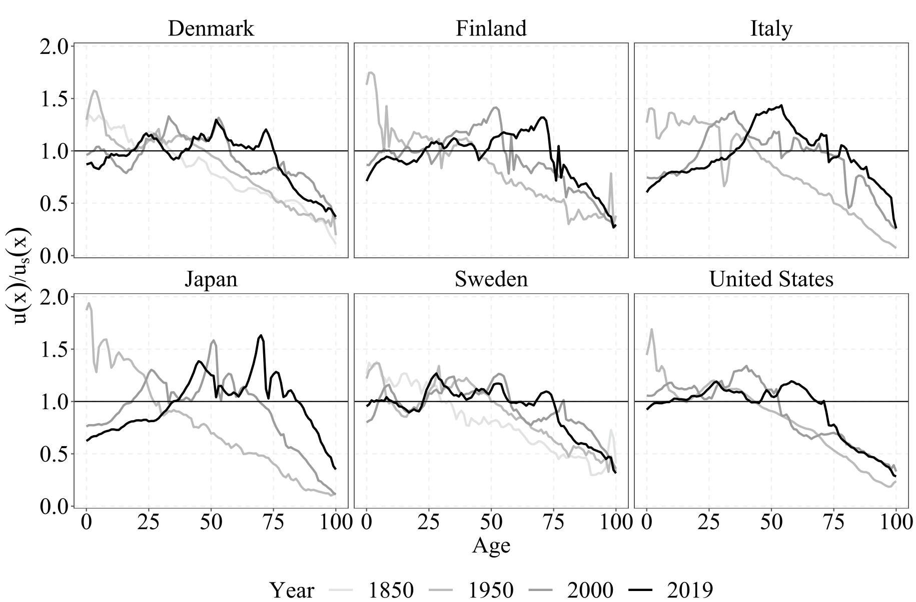

Fig. 8 Ratio of the age distributions of the observed and stationary population in 6 countries from HMD, 1850–2019

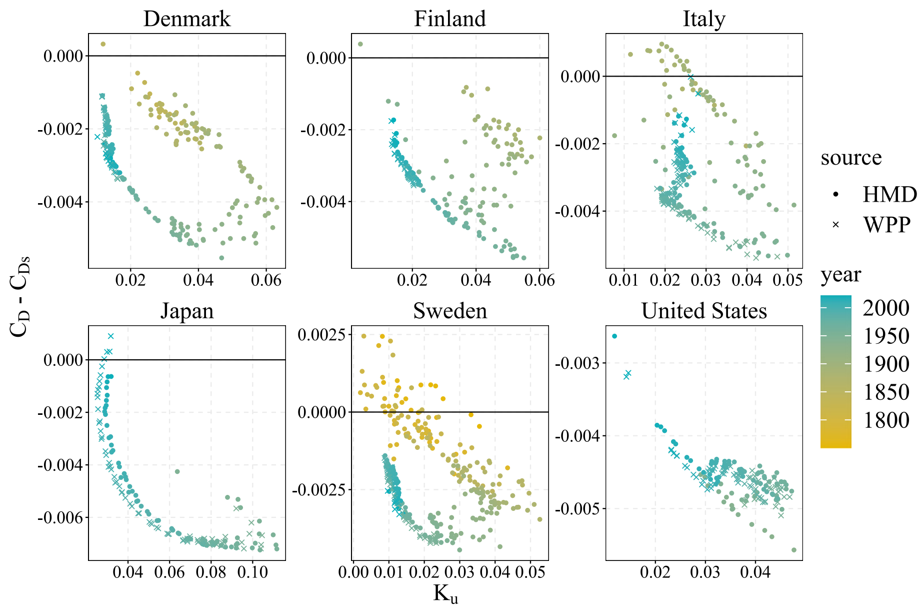

Fig. 9 The deviation (K_u) and the sensitivity (C_D—C_{Ds}) in 6 countries from HMD and WPP, 1751–2021

(and so larger in magnitude) as the distance K_u decreases. A similar negative correlation is seen in for many years in the long-term data for Sweden. Focusing on the period 1900 to 1970, Fig. 3 shows that C_D falls relative to the stationary C_{Ds}, while over the same period Fig. 7 shows that the distance K_u decreases.

Finally here we note that the distance K_u is the main driver of the crude death rate (as compared with shifts in the age-pattern of mortality). Thus, there is relatively high discrimination of the deviation for distinguishing the mismatch pattern ("actual deaths > expected deaths" and "actual deaths < expected deaths"), as the corresponding area under the receiver operating characteristic curve (AUC) for K_u is 0.74 in HMD and 0.7 in WPP data.

Stable vs. stationary populations

Much interest in demography has been devoted to stationary populations (Coale, 1972; Keyfitz, 1965). McCann (1976) analyzed crude death rate in stable populations growing at some constant rate, distinct from the stationary populations we consider here. Given that populations are rarely stable, and indeed the demographic transition means that growth rates must change with time, analyses based on stable growth are unlikely to be useful here. Even so, any period fertility and mortality leads to a growth rate and a purely hypothetical stable population. So we asked whether that stable age-distribution could be used instead of the stationary distribution. The answer was negative, but we present the comparison in the Appendix.

Discussion

The over-prediction of the crude death rate in terms of life expectancy at birth is widely observed in practice and is puzzling for non-demographers and demographers alike. Here we explored this discrepancy in several ways. We showed analytically that whenever a population age distribution is close to stationary, the stationary population has many more deaths than the actual number. Secondly, our formulas measure precisely the age-specific sensitivity of the crude death rate. In this context, we mention also the more general results in the interesting papers by Aburto et al. (2020), Vaupel (2021) and Nigri et al. (2022).

We used a large data set and found that over-prediction of crude death rate is widespread. The deviation between the observed and stationary populations is an important driver of the mismatch in the actual and expected number of deaths. We present both short-term (decades) and long-term (centuries) of analysis suggesting that the demographic transition produced persistent bulges at ages where the mortality rates are lower. The difference between actual deaths and stationary deaths is mainly driven by the central age groups. This is interesting since this means that more people than we might expect (under stationarity or stability) arrive at adult age. These factors help explain why we find widespread over-prediction of the actual number of deaths.

Appendix

A different distance measure

We used the Kullback–Leibler distance in the main text. But as noted, qualitatively similar results are obtained using an alternative indicator:

$$S_u = \int dx\,(\,u(x)\ -u_s(x)\,)^2\,,$$

and are given below.

Figure 10 shows the distribution of the distances. Distances over time are shown for 3 countries in Fig. 11. And the long-term pattern for Sweden is shown in Fig. 12.

Finally, Fig. 13 shows the over-prediction as a function of the distance S_u for 6 countries.

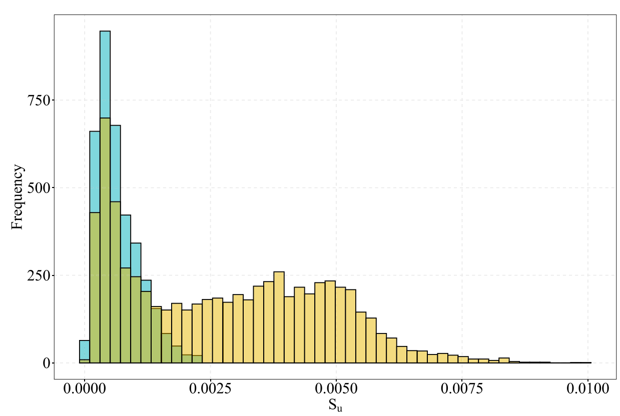

Fig. 10 Frequency of S_u value in countries from HMD and WPP. HMD data in green (or blue) and WPP data in yellow

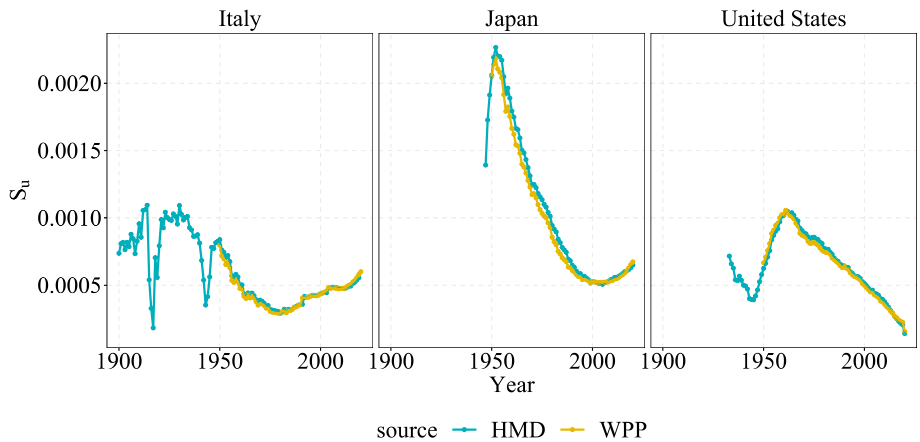

Fig. 11 S_u value in 3 countries from HMD and WPP, 1900–2020

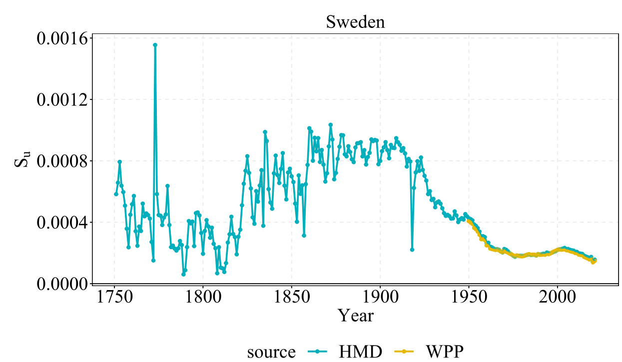

Fig. 12 S_u value in Sweden from HMD and WPP, 1751–2021

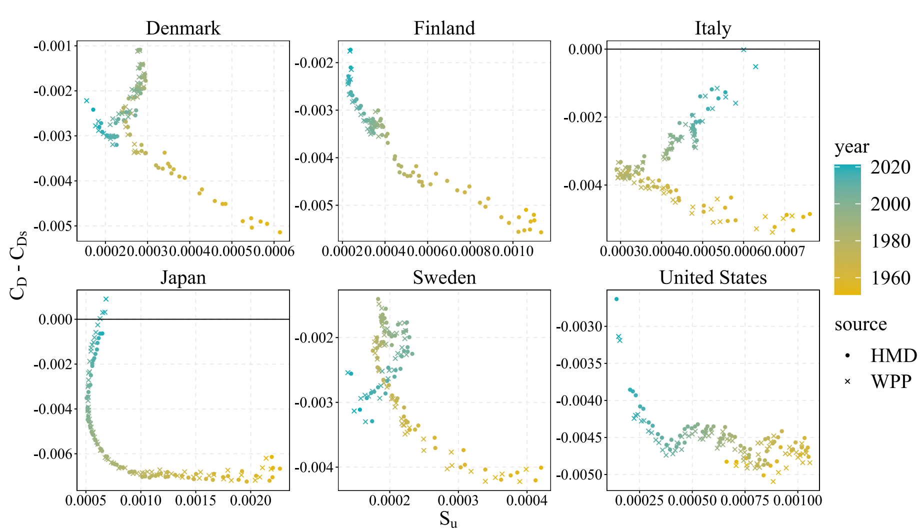

Fig. 13 The deviation (S_u) and the sensitivity (C_D—C_{Ds}) in 6 countries from HMD and WPP, 1751–2021

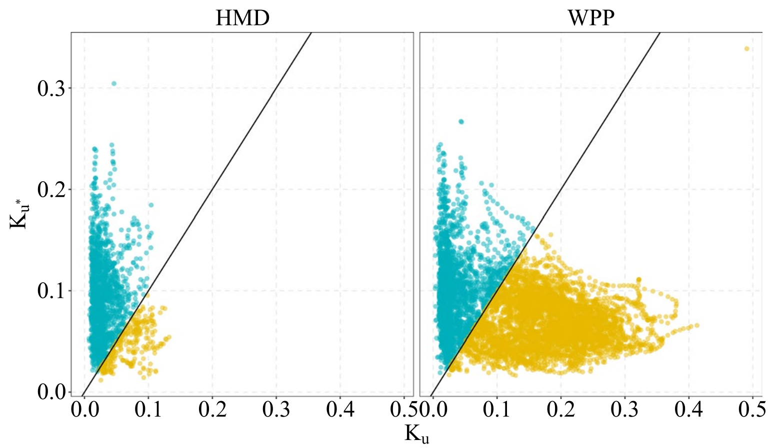

Fig. 14 K_u and K_u^* value in countries from HMD and WPP

Stable populations

We have seen that populations are generally non-stationary and we know how that difference will affect the crude death rate. But of course, much formal demography focuses on stability rather than stationarity. So we asked whether populations were in fact closer to the stable structure implied by their fertility and mortality rates, rather than to the corresponding stationary structure. Using fertility as well as mortality yielded a corresponding stable population growth rate, and then a stable population structure $u_s^*(x)$. The distance from the actual population structure to that corresponding stable age distribution is

$$K_u^* = -\int dx\; u_s^*(x) \log\left(\frac{u(x)}{u_s^*(x)}\right) \geq 0\,. \tag{17}$$

We compare the distances K_u^* and K_u for both HMD and WPP, in Fig. 14. For the HMD data, we used corresponding fertility data from the Human Fertility Database (Max Planck Institute for Germany & Vienna Institute of Demography Austria, 2022). As we expect both distances are usually non-zero, because actual populations are non-stable and non-stationary. For the HMD countries, most of the distances K_u^* are greater than the distances K_u values, so the deviation from stability is generally larger than the deviation from stationarity. However, the WPP values show the opposite pattern. We do not understand why.

Abbreviations

WPP World Population Prospects
HMD Human Mortality Database
AUC Area under the receiver operating characteristic curve

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Author contributions

HL performed the analysis, wrote the first draft, and edited the manuscript. ST developed the idea, guided the writing of the paper, and commented on and revised the manuscript. ZG performed some analysis and commented on the manuscript. All authors read and approved the final manuscript.

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Availability of data and materials
Data are available in the World Population Prospects (https://population.un.org/wpp/), the Human Mortality Database (https://www.mortality.org/), and the Human Fertility Database (https://www.humanfertility.org/).

Declarations

Competing interests
The authors declare that they have no competing interests.

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References
Aburto, J. M., Villavicencio, F., Basellini, U., Kjærgaard, S., & Vaupel, J. W. (2020). Dynamics of life expectancy and life span equality. *Proceedings of the National Academy of Sciences., 117*(10), 5250–5259.
Barbieri, M., Wilmoth, J. R., Shkolnikov, V. M., Glei, D., Jasilionis, D., Jdanov, D., Boe, C., Riffe, T., Grigoriev, P., & Winant, C. (2015). Data resource profile: The human mortality database (HMD). *International Journal of Epidemiology., 44*, 5.
Berkeley USA University of California and Max Planck Institute for Germany. (2022). Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org or www.humanmortality.de.
Coale, A. J. (1972). *The growth and structure of human populations: A mathematical investigation*. Princeton University Press.
Demetrius, L. (1978). Adaptive value, entropy and survivorship curves. *Nature, 275*, 5677.
Department of Economic United Nations and Population Division Social Affairs. (2022). World Population Prospects. United Nations, Department of Economic and Social Affairs, Population Division. Available at https://population.un.org/wpp/.
Glei, D. A., Borges, G., Riffe, T., Andreeva, M., & Menares, F. (2015). About mortality data for Italy. Human Mortality Database. Italy. Available at https://www.mortality.org/File/GetDocument/hmd.v6/ITA/Public/InputDB/ITAcom.pdf.
Goldman, N., & Lord, G. (1986). A new look at entropy and the life table. *Demography, 23*, 2.
Keyfitz, N. (1965). The intrinsic rate of natural increase and the dominant root of the projection matrix. *Population Studies., 18*, 3.
Keyfitz, N. (1977). *Introduction to the mathematics of population with revisions*. Menlo Park, CA: Addison-Wesley Publishing Company.
Kullback, S., & Leibler, R. A. (1951). On information and sufficiency. *The Annals of Mathematical Statistics., 22*, 1.
Max Planck Institute for Germany and Vienna Institute of Demography Austria.(2022). Human Fertility Database, Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at www.humanfertility.org.
McCann, J. C. (1976). A technique for estimating life expectancy with crude vital rates. *Demography, 13*, 2.
Nigri, A., Barbi, E., & Levantesi, S. (2022). The relationship between longevity and lifespan variation. *Statistical Methods Applications., 31*(3), 1–13.
United Nations Development Programme. (2022). Human Development Report 2021–22. United Nations Development Programme. Available at https://hdr.undp.org/system/files/documents/global-report-document/hdr2021-22pdf_1.pdf.
Vaupel, J. W. (1986). How change in age-specific mortality affects life expectancy. *Population Studies., 40*, 1.
Vaupel, J. W., Villavicencio, F., & Bergeron-Boucher, M. P. (2021). Demographic perspectives on the rise of longevity. *Proceedings of the National Academy of Sciences, 118*(9), e2019536118. https://doi.org/10.1073/pnas.2019536118

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